CBSE Class 12 Mathematics

Important Questions and Answers for Board Exams

Printable question bank: important, previous-year, case-based, application and model questions with answers and worked solutions

How to use this sheet. Try every question on paper before you read the answer. Important Questions are chapter-wise favourites; Previous Year Questions are from the March 2026 board paper (Code 65/1/1); Model Questions are practice questions, *not* predictions of what will appear. Bold letters such as **a** and **b** are vectors; $\tan^{-1}$ is inverse tangent. Please confirm the final syllabus on the official CBSE academic website (cbseacademic.nic.in).

PART 1 · IMPORTANT QUESTIONS WITH ANSWERS

Unit I: Relations, Functions and Inverse Trigonometric Functions

Q1. Check whether the relation R = {(a, b) : a ≤ b²} on the set of real numbers is reflexive, symmetric or transitive.

Ans. It is none of the three. Not reflexive, because (1/2, 1/2) ∉ R (1/2 is not ≤ 1/4). Not symmetric, because (1, 4) ∈ R but (4, 1) ∉ R. Not transitive, because (3, 2) and (2, 1.5) are in R, but (3, 1.5) is not.

Q2. Show that f : R → R given by f(x) = 3 − 4x is one-one and onto.

Ans. If $f(x_1) = f(x_2)$, then $3 - 4x_1 = 3 - 4x_2$, so $x_1 = x_2$ (one-one). For any y in R, x = (3 − y)/4 is a real number with f(x) = y (onto). So f is bijective.

Q3. Find the principal value of $\sin^{-1}(-1/2) + \cos^{-1}(-1/2)$.

Ans. $-\pi/6 + 2\pi/3 = \pi/2$.

Q4. Prove that $2\tan^{-1}(1/2) + \tan^{-1}(1/7) = \tan^{-1}(31/17)$.

Ans. Proved. (Full working is in Part 6, Worked Solutions.)

Unit II: Algebra

Q5. Find the inverse of A = [1 2; 3 4] (rows separated by a semicolon).

Ans. |A| = −2, so $A^{-1} = (1/-2)[4\ -2; -3\ 1] = [-2\ 1; 3/2\ -1/2]$.

Q6. Express A = [3 5; 1 −1] as the sum of a symmetric and a skew-symmetric matrix.

Ans. Symmetric part P = (A + A′)/2 = [3 3; 3 −1]. Skew-symmetric part Q = (A − A′)/2 = [0 2; −2 0]. And A = P + Q.

Q7. Using determinants, find the area of the triangle with vertices (1, 0), (6, 0) and (4, 3).

Ans. Area = ½ |1(0 − 3) + 6(3 − 0) + 4(0 − 0)| = ½ × 15 = 15/2 square units.

Q8. Solve using the matrix method: x + 2y = 4 and 3x − y = 5.

Ans. x = 2, y = 1. (Check: 2 + 2 = 4 and 6 − 1 = 5.)

Unit III: Calculus

Q9. Differentiate $\sin^{-1}(2x\sqrt{(1 - x^2)})$ with respect to x, for $-1/\sqrt{2} < x < 1/\sqrt{2}$.

Ans. Put x = sin θ. The expression becomes $\sin^{-1}(\sin 2\theta) = 2\theta = 2\sin^{-1}x$, so the derivative is $2/\sqrt{(1 - x^2)}$.

Q10. Find dy/dx if $y = x^x$.

Ans. Take logs: $\log y = x \log x$. Differentiating, $dy/dx = x^x(1 + \log x)$.

Q11. If $x = a(\theta - \sin\theta)$ and $y = a(1 - \cos\theta)$, find dy/dx.

Ans. $dx/d\theta = a(1 - \cos\theta)$ and $dy/d\theta = a \sin\theta$, so $dy/dx = \sin\theta/(1 - \cos\theta) = \cot(\theta/2)$.

Q12. Find the intervals in which $f(x) = 2x^3 - 9x^2 + 12x + 15$ is increasing or decreasing.

Ans. $f'(x) = 6(x - 1)(x - 2)$. The function is increasing on $(-\infty, 1)$ and $(2, \infty)$, and decreasing on $(1, 2)$.

Q13. Find the local maximum and local minimum values of $f(x) = x^3 - 3x$.

Ans. $f'(x) = 3x^2 - 3 = 0$ gives $x = \pm 1$. Since $f''(x) = 6x$, there is a local maximum value 2 at $x = -1$ and a local minimum value -2 at $x = 1$.

Q14. Evaluate $\int x\, e^x\, dx$.

Ans. Using integration by parts: $e^x(x - 1) + C$.

Q15. Evaluate $\int dx / ((x + 1)(x + 2))$.

Ans. $1/((x + 1)(x + 2)) = 1/(x + 1) - 1/(x + 2)$, so the integral is $\log|(x + 1)/(x + 2)| + C$.

Q16. Evaluate $\int$ from 0 to $\pi/2$ of $\sin x/(\sin x + \cos x)\, dx$.

Ans. Use the property $\int f(x)dx = \int f(a - x)dx$. Adding the two forms gives $2I = \pi/2$, so $I = \pi/4$.

Q17. Using integration, find the area of the ellipse $x^2/a^2 + y^2/b^2 = 1$.

Ans. Area $= 4 \int$ from 0 to a of $(b/a)\sqrt{(a^2 - x^2)}\, dx = \pi ab$ square units.

Q18. Solve $dy/dx + 2y = \sin x$.

Ans. The integrating factor is e^{2x}. The general solution is $y = (2 \sin x - \cos x)/5 + C\, e^{-2x}$.

Unit IV: Vectors and Three-Dimensional Geometry

Q19. Find the shortest distance between the lines $\mathbf{r} = (\mathbf{i} + 2\mathbf{j} + \mathbf{k}) + \lambda(\mathbf{i} - \mathbf{j} + \mathbf{k})$ and $\mathbf{r} = (2\mathbf{i} - \mathbf{j} - \mathbf{k}) + \mu(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$.

Ans. $3/\sqrt{2}$ units. (Full working is in Part 6, Worked Solutions.)

Q20. If a line makes angles α, β, γ with the coordinate axes, prove that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.

Ans. Since $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, we get $(1 - \cos^2\alpha) + (1 - \cos^2\beta) + (1 - \cos^2\gamma) = 3 - 1 = 2$.

Unit V: Linear Programming

Q21. Maximise $Z = 3x + 4y$ subject to $x + y \le 4$, $x \ge 0$, $y \ge 0$.

Ans. The corner points are (0, 0), (4, 0) and (0, 4), giving Z = 0, 12 and 16. The maximum value is 16 at (0, 4).

Unit VI: Probability

Q22. Machines A and B produce 60% and 40% of a factory's items. Of their output, 2% and 3% are defective respectively. An item picked at random is defective. What is the probability that it came from machine B?

Ans. 1/2. (Full working is in Part 6, Worked Solutions.)

PART 2 · PREVIOUS YEAR QUESTIONS (March 2026, Code 65/1/1)

PYQ 1 · 2026 · Section A (1 mark). The least value of $f(x) = x^3 - 12x$, $x \in [0, 3]$ is: (A) -16 (B) -9 (C) 0 (D) 16

Ans. (A) -16. ($f'(x) = 0$ at $x = 2$; $f(0) = 0$, $f(2) = -16$, $f(3) = -9$.)

PYQ 2 · 2026 · Section A (1 mark). The value of $\int$ from -1 to 1 of $x^3/(x^2 + 2|x| + 1)\,dx$ is: (A) 0 (B) log 2 (C) 2 log 2 (D) ½ log 2

Ans. (A) 0. The integrand is an odd function over a symmetric interval.

PYQ 3 · 2026 · Section A (1 mark). If $|\mathbf{a}| = 8$, $|\mathbf{b}| = 3$ and $|\mathbf{a} \times \mathbf{b}| = 12$, then $|\mathbf{a} \cdot \mathbf{b}|$ is: (A) $6\sqrt{3}$ (B) $8\sqrt{3}$ (C) $12\sqrt{3}$ (D) $3\sqrt{12}$

Ans. (C) $12\sqrt{3}$. ($|\mathbf{a} \times \mathbf{b}|^2 + |\mathbf{a} \cdot \mathbf{b}|^2 = |\mathbf{a}|^2|\mathbf{b}|^2$, so $|\mathbf{a} \cdot \mathbf{b}|^2 = 576 - 144 = 432$.)

PYQ 4 · 2026 · Section A, Assertion-Reason (1 mark). *Assertion (A):* In throwing an unbiased die, the probability of getting a prime number given that the number is odd is 2/3. *Reason (R):* For any two events A and B, $P(A|B) = P(A \cup B)/P(B)$.

Ans. (C) A is true and R is false. Among odd numbers {1, 3, 5}, the primes are {3, 5}, so the probability is 2/3. The correct formula uses $P(A \cap B)$, not $P(A \cup B)$.

PYQ 5 · 2026 · Section B (2 marks). Check whether $f(x) = |x - 3|/(2(x - 3))$ for $x < 3$, and $f(x) = (x - 6)/6$ for $x \geq 3$, is continuous at $x = 3$.

Ans. For $x < 3$, $f(x) = -1/2$, so LHL $= -1/2$. RHL $= (3 - 6)/6 = -1/2$ and $f(3) = -1/2$. All three match, so f is continuous at $x = 3$.

PYQ 6 · 2026 · Section B (2 marks). A room freshener bottle shaped like an inverted cone loses perfume at a steady 1 mm^3/min. Find the rate at which the level is dropping when the level is 10 mm, if the semi-vertical angle is $\pi/6$.

Ans. $-3/(100\pi)$ mm/min. (Full working is in Part 6, Worked Solutions.)

PYQ 7 · 2026 · Section B (2 marks). Find the vector of magnitude 14 in the direction of QP, where P = (1, 3, 2) and Q = (−1, 0, 8).

Ans. $\mathrm{QP} = 2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$, with magnitude 7. The required vector is $2 \times \mathrm{QP} = 4\mathbf{i} + 6\mathbf{j} - 12\mathbf{k}$.

PYQ 8 · 2026 · Section C (3 marks). Evaluate $\int$ from 0 to 1 of $x \tan^{-1}x\,dx$.

Ans. $\pi/4 - 1/2$. (Full working is in Part 6, Worked Solutions.)

PYQ 9 · 2026 · Section C (3 marks). Find the general solution of $x^2\,(dy/dx) = x^2 + xy + y^2$.

Ans. $\tan^{-1}(y/x) = \log|x| + C$. It's a homogeneous equation: put $y = vx$, which gives $dv/(1 + v^2) = dx/x$.

PYQ 10 · 2026 · Section C (3 marks). Bag I has 3 red and 4 white balls; Bag II has 8 red and 6 white balls. A die is thrown. If the number is less than 3, a ball is drawn from Bag I, otherwise from Bag II. Find the probability that the ball drawn is red.

Ans. $(2/6)(3/7) + (4/6)(8/14) = 1/7 + 8/21 = 11/21$.

PYQ 11 · 2026 · Section D (5 marks). A relation R on Z is defined by $R = \{(x, y) : |x - y|$ is divisible by a prime number p$\}$. Check whether R is an equivalence relation.

Ans. Yes. *Reflexive:* $|x - x| = 0$ is divisible by p. *Symmetric:* $|x - y| = |y - x|$. *Transitive:* if $x - y$ and $y - z$ are divisible by p, so is their sum $x - z$. All three properties hold, so R is an equivalence relation.

PYQ 12 · 2026 · Section D (5 marks). Check whether the lines $(x - 1)/2 = (y - 2)/3 = (z - 3)/4$ and $(x - 4)/5 = (y - 1)/2 = z$ are parallel. If parallel, find the distance between them; otherwise find the point of intersection, if they intersect.

Ans. Not parallel; they intersect at (−1, −1, −1). (Full working is in Part 6, Worked Solutions.)

PART 3 · COMPETENCY-BASED AND CASE-BASED QUESTIONS

Underneath the story there is almost always one familiar idea (maximum and minimum, area under a curve, probability). Read slowly and write down what the question is really asking for.

Case Study 1 · 2026 Board Paper, Section E (4 marks)

An online delivery company in a city has 5000 subscribers, each paying Rs. 300 a year for unlimited free deliveries. The company wants to raise the annual fee. It is predicted that for every increase of Rs. 1, ten subscribers will discontinue. Assume the fee is increased by Rs. x.

(i) How many subscribers will discontinue after an increase of Rs. x? *(1 mark)*

Ans. 10x subscribers.

(ii) If R(x) is the total revenue after the increase, express R(x) as a function of x. *(1 mark)*

Ans. $R(x) = (5000 - 10x)(300 + x)$.

(iii)(a) Find the value of x for which R(x) is maximum. *(2 marks)*

Ans. $R(x) = 15{,}00{,}000 + 2000x - 10x^2$, so $R'(x) = 2000 - 20x$. Setting $R'(x) = 0$ gives $x = 100$. Since $R''(x) = -20 < 0$, revenue is maximum at $x = 100$.

OR (iii)(b) Find the sub-intervals of (0, 5000) in which R(x) is increasing and decreasing. *(2 marks)*

Ans. $R'(x) > 0$ on (0, 100), so R is increasing there. $R'(x) < 0$ on (100, 5000), so R is decreasing there.

Case Study 2 · Practice Question

A school runs two buses, A and B. Bus A carries 60% of the students and Bus B carries 40%. The probability that a student travelling by Bus A reaches late is 0.05, and by Bus B is 0.10.

(i) Find the probability that a randomly chosen student travels by Bus A and reaches late. *(1 mark)*

Ans. $0.6 \times 0.05 = 0.03$.

(ii) Find the probability that a randomly chosen student reaches late. *(1 mark)*

Ans. $0.6 \times 0.05 + 0.4 \times 0.10 = 0.03 + 0.04 = 0.07$.

(iii) A student is found to have reached late. Find the probability that the student travelled by Bus B. *(2 marks)*

Ans. 0.04/0.07 = 4/7.

Case Study 3 · Practice Question

A bridge has a parabolic arch whose shape is given by $y = 4 - x^2$ (in metres), with the ground along the x-axis. The arch is to be painted on its front face, and painting costs Rs. 150 per square metre.

(i) At which points does the arch meet the ground? *(1 mark)*

Ans. Put $y = 0$: $x = \pm 2$. The points are $(-2, 0)$ and $(2, 0)$.

(ii) Write the definite integral that gives the area of the front face. *(1 mark)*

Ans. Area = $\int$ from -2 to 2 of $(4 - x^2)\,dx$.

(iii) Find the area and the total cost of painting. *(2 marks)*

Ans. Area $= 2\int$ from 0 to 2 of $(4 - x^2)\,dx = 2[4x - x^3/3]$ from 0 to $2 = 2(8 - 8/3) = 32/3$ sq m. Cost $= 32/3 \times 150 =$ Rs. 1,600.

PART 4 · THREE QUICK APPLICATION QUESTIONS (PRACTICE)

A1. A 5 m ladder leans against a wall. Its foot is pulled away from the wall at 2 m/s. How fast is the top sliding down when the foot is 3 m from the wall?

Ans. $x^2 + y^2 = 25$. When $x = 3$, $y = 4$. Differentiating, $x(dx/dt) + y(dy/dt) = 0$, so $dy/dt = -(3/4)(2) = -1.5$ m/s. The top slides down at 1.5 m/s.

A2. A farmer has 200 m of fencing and wants to enclose a rectangular field along a straight river, so only three sides need fencing. What dimensions give the maximum area?

Ans. Let the two sides perpendicular to the river be x and the side parallel to it be y, so $2x + y = 200$. Area $A = x(200 - 2x)$. Then $dA/dx = 200 - 4x = 0$ gives $x = 50$, $y = 100$, and $d^2A/dx^2 = -4 < 0$. Maximum area = 5000 sq m.

A3. A drone flies from A(1, 2, 3) to B(4, 6, 15). Find the distance covered and the direction cosines of its path.

Ans. $AB = 3\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}$, so the distance is $\sqrt{(9 + 16 + 144)} = 13$ units. The direction cosines are 3/13, 4/13 and 12/13.

PART 5 · MODEL QUESTIONS (PRACTICE ONLY)

These are practice questions in board-exam style. They are **not** predicted or guaranteed board questions. Set a timer, write full steps, and check against the answer key below.

M1 (1 mark). If A is a 3 × 3 matrix with $|A| = 4$, then $|2A|$ equals: (A) 8 (B) 16 (C) 32 (D) 64

M2 (1 mark). The value of $\int$ from $-\pi/2$ to $\pi/2$ of $\sin^3 x\, dx$ is: (A) 0 (B) 2/3 (C) 4/3 (D) 1

M3 (1 mark, Assertion-Reason). *Assertion (A):* f(x) = |x| is continuous at x = 0. *Reason (R):* f(x) = |x| is differentiable at x = 0. Choose: (A) Both true, R explains A. (B) Both true, R does not explain A. (C) A true, R false. (D) A false, R true.

M4 (2 marks). If $y = \log(\sec x + \tan x)$, find dy/dx.

M5 (2 marks). Find the value of k for which $f(x) = kx + 1$ $(x \le 5)$, $f(x) = 3x - 5$ $(x > 5)$ is continuous at x = 5.

M6 (3 marks). Evaluate $\int dx / (x(x^2 + 1))$.

M7 (3 marks). Solve the differential equation $dy/dx = (x + y)/x$.

M8 (3 marks). Find the area of the region bounded by the parabola $y^2 = 4x$ and the line x = 3.

M9 (5 marks). Solve the following system using the matrix method: $x - y + 2z = 7$, $3x + 4y - 5z = -5$, $2x - y + 3z = 12$.

M10 (5 marks). Solve graphically: Minimise $Z = 3x + 5y$ subject to $x + 3y \ge 3$, $x + y \ge 2$, $x \ge 0$, $y \ge 0$.

PART 6 · ANSWER EXPLANATIONS (WORKED SOLUTIONS)

Boards reward *steps*, not just answers. These are fully worked solutions for some of the questions above, written the way you would present them in the answer sheet.

Important Q4: Prove $2\tan^{-1}(1/2) + \tan^{-1}(1/7) = \tan^{-1}(31/17)$

Step 1. Use $2\tan^{-1}x = \tan^{-1}(2x/(1 - x^2))$ for $|x| < 1$. With $x = 1/2$: $2x = 1$ and $1 - x^2 = 3/4$, so $2\tan^{-1}(1/2) = \tan^{-1}(4/3)$.

Step 2. Use $\tan^{-1}a + \tan^{-1}b = \tan^{-1}((a + b)/(1 - ab))$, valid when $ab < 1$. Here $a = 4/3$ and $b = 1/7$, so $ab = 4/21 < 1$.

Step 3. $a + b = 28/21 + 3/21 = 31/21$, and $1 - ab = 17/21$. So the sum is $\tan^{-1}((31/21)/(17/21)) = \tan^{-1}(31/17)$. Hence proved.

Important Q19: Shortest distance between skew lines

Step 1. Read off the points and directions. $a_1 = (1, 2, 1)$, $\mathbf{b}_1 = (1, -1, 1)$; $a_2 = (2, -1, -1)$, $\mathbf{b}_2 = (2, 1, 2)$.

Step 2. Find $\mathbf{b}_1 \times \mathbf{b}_2 = (-1\cdot2 - 1\cdot1)\mathbf{i} - (1\cdot2 - 1\cdot2)\mathbf{j} + (1\cdot1 - (-1)\cdot2)\mathbf{k} = -3\mathbf{i} + 0\mathbf{j} + 3\mathbf{k}$. Its magnitude is $\sqrt{(9 + 9)} = 3\sqrt{2}$.

Step 3. $a_2 - a_1 = (1, -3, -2)$. Its dot product with $(-3, 0, 3)$ is $-3 + 0 - 6 = -9$.

Step 4. Shortest distance $= |-9| / (3\sqrt{2}) = 3/\sqrt{2}$ units.

Important Q22: Probability using Bayes' theorem

Step 1. Define events. Let A = item made by machine A, B = item made by machine B, D = item is defective. $P(A) = 0.6$, $P(B) = 0.4$, $P(D|A) = 0.02$, $P(D|B) = 0.03$.

Step 2. Total probability: $P(D) = 0.6 \times 0.02 + 0.4 \times 0.03 = 0.012 + 0.012 = 0.024$.

Step 3. Bayes' theorem: $P(B|D) = P(B)P(D|B)/P(D) = 0.012/0.024 = 1/2$.

PYQ 6 (2026): Cone and rate of change

Step 1. Let r be the radius and h the height of the perfume surface. Given $dV/dt = -1$ mm³/min. Because the semi-vertical angle is $\pi/6$, $r/h = \tan(\pi/6) = 1/\sqrt{3}$, so $r = h/\sqrt{3}$.

Step 2. $V = (1/3)\pi r^2 h = (1/3)\pi(h^2/3)h = \pi h^3/9$.

Step 3. Differentiate: $dV/dt = (\pi h^2/3)(dh/dt)$.

Step 4. At $h = 10$: $-1 = (\pi \times 100/3)(dh/dt)$, so $dh/dt = -3/(100\pi)$ mm/min. The negative sign tells us the level is falling.

PYQ 8 (2026): $\int$ from 0 to 1 of $x\tan^{-1}x\,dx$

Step 1. Integrate by parts, taking $\tan^{-1}x$ as the first function (ILATE) and x as the second: $\int x\tan^{-1}x\,dx = (x^2/2)\tan^{-1}x - \int x^2/(2(1 + x^2))\,dx$.

Step 2. At the limits, the first term gives $(1/2)(\pi/4) - 0 = \pi/8$.

Step 3. Rewrite $x^2/(1 + x^2) = 1 - 1/(1 + x^2)$. So the second part is $(1/2)[x - \tan^{-1}x]$ from 0 to 1 $= (1/2)(1 - \pi/4) = 1/2 - \pi/8$.

Step 4. Result $= \pi/8 - (1/2 - \pi/8) = \pi/4 - 1/2$.

PYQ 12 (2026): Are the lines parallel, and if not, do they meet?

Step 1. Direction ratios are (2, 3, 4) and (5, 2, 1). They are not proportional, so the lines are not parallel.

Step 2. A general point on the first line is $(1 + 2\lambda, 2 + 3\lambda, 3 + 4\lambda)$; on the second, $(4 + 5\mu, 1 + 2\mu, \mu)$.

Step 3. Equate the x and y coordinates: $2\lambda - 5\mu = 3$ and $3\lambda - 2\mu = -1$. Solving, $\mu = -1$ and $\lambda = -1$.

Step 4. Check the z coordinates: $3 + 4\lambda = -1$ and $\mu = -1$. They match, so the lines intersect. Putting $\lambda = -1$ gives the point $(-1, -1, -1)$.

PART 7 · MODEL QUESTIONS: ANSWER KEY

Question	Answer
M1	(C) 32, because $\|2A\| = 2^3\|A\| = 8 \times 4 = 32$
M2	(A) 0, because $\sin^3 x$ is an odd function
M3	(C), A is true and R is false ($\|x\|$ is not differentiable at $x = 0$)
M4	sec x
M5	k = 9/5 (from $5k + 1 = 10$)
M6	$\log\|x\| - \frac{1}{2}\log(x^2 + 1) + C$
M7	$y = x(\log\|x\| + C)$
M8	$8\sqrt{3}$ square units
M9	x = 2, y = 1, z = 3 (working below)
M10	Minimum Z = 7 at (3/2, 1/2) (working below)

Note: in the source article these cells of the answer key were cut short; M1, M3 and M6 are completed here by working them out, and M7 follows the article's own "in brief" working below.

M7 in brief

The equation is homogeneous. Put $y = vx$, so $v + x(dv/dx) = 1 + v$, which gives $dv = dx/x$. Integrating, $v = \log|x| + C$, so $y = x(\log|x| + C)$.

M8 in brief

By symmetry about the x-axis, area $= 2 \int$ from 0 to 3 of $2\sqrt{x}\,dx = 4 \times (2/3) \times 3^{3/2} = 8\sqrt{3}$.

M9: Matrix method, step by step

Write the system as $AX = B$, with $A = [1\ -1\ 2;\ 3\ 4\ -5;\ 2\ -1\ 3]$, $X = (x, y, z)$ and $B = (7, -5, 12)$.

Step 1. $|A| = 1(12 - 5) + 1(9 + 10) + 2(-3 - 8) = 7 + 19 - 22 = 4$. Since $|A| \neq 0$, A^{-1} exists and the solution is unique.

Step 2. Cofactors: $C_{11} = 7$, $C_{12} = -19$, $C_{13} = -11$, $C_{21} = 1$, $C_{22} = -1$, $C_{23} = -1$, $C_{31} = -3$, $C_{32} = 11$, $C_{33} = 7$. The adjoint is the transpose: adj $A = [7\ 1\ -3;\ -19\ -1\ 11;\ -11\ -1\ 7]$.

Step 3. $X = A^{-1}B = (1/4)(\text{adj } A)B$. Multiplying: $7(7) + 1(-5) - 3(12) = 8$; $-19(7) - 1(-5) + 11(12) = 4$; $-11(7) - 1(-5) + 7(12) = 12$.

Step 4. So $x = 8/4 = 2$, $y = 4/4 = 1$, $z = 12/4 = 3$. Quick check in the first equation: $2 - 1 + 6 = 7$. ✓

M10: Linear programming, step by step

Step 1. Draw the lines $x + 3y = 3$ and $x + y = 2$ and shade the region where both inequalities ($\geq$) hold in the first quadrant. The region is unbounded.

Step 2. Find the corner points: (0, 2), (3, 0), and the intersection of the two lines, which is (3/2, 1/2).

Step 3. Evaluate $Z = 3x + 5y$: at (0, 2) it is 10, at (3, 0) it is 9, and at (3/2, 1/2) it is 7.

Step 4. Because the region is unbounded, check that the open half-plane $3x + 5y < 7$ has no point in common with the feasible region. It doesn't, so the minimum value of Z is 7, attained at (3/2, 1/2). Don't skip this check; it's worth a mark.